Options Analysis

Guilherme Salomé

November 29, 2018

The basic ideas of options are very simple so long as one does not bring in a lot of extraneous material. At the close of trading on Friday, October 14, the stock Microsoft (MSFT) closed at \$ 57.42; that meant that Friday afternoon an investor who was optimistic about MSFT could have purchased the stock at about that price.

A typical personal investment of 100 shares would entail \$ 5,742 plus commissions. At the same time, the call option to buy MSFT at \$ 57.50 with expiration Dec 12, 2016 closed at \$ 1.85. Why would anyone have paid more (called the option premium), i.e., \$ 1.85, for the right to buy MSFT in mid-Dec at \$ 57.50 when it could have been purchased at essentially the same price right then? The answer is obvious.

If MSFT takes off to, say, \$ 62.50 the holder of the option essentially makes a profit of \$ 5.00 on a \$ 1.85 investment. On the other hand, if bad news sinks MSFT to below \$ 57.50 then the option expires worthlessly and the holder is only out the \$ 1.85 paid initially. Clearly, the option creates an interesting non-linear payoff.

Volatility is the source of value of the option. If there were no uncertainty, then this option as described would be worthless. Note that this option is essentially right "at the money", in that the strike price of \$ 57.50 is essentially the same as the stock's current price. If the option were "in the money", that is the stock price exceeded the strike price, and there were no volatility then the option would be worth just the difference between the prices (adjusted for the interest between now and December). Whereas, if the option were "out of the money", that is the stock price was less than the strike price, it would be worthless. But we know there is volatility or uncertainty, and we have been studying it all semester.

1 Black-Scholes Model

The classical Black-Scholes (BLS) model provides a means to compute the "Fair Value" of an option. The model assumes the semimartingale model we use with the log of the stock price (here called S) following

$$dX_t \;=\; \mu\, dt \;+\; \sqrt{c}\, dW_t$$

where $X = \log(S)$. The model above is what we have been using all semester except for the assumption that $c = \sigma^2$ is constant. Let T denote the expiration time with now being $t = 0$, and let K be the strike price of the option. Then, the value of the option at expiration is

$$\text{Payoff of call option at } T\text{: } \max(S_T - K, 0)$$

The dynamics of the log-price imply

$$\log(S_T) \sim N(\log(S_0) + \mu T, \sigma^2 T)$$

Under these conditions, the BLS fair value is just the present value of the expected payoff

$$\mathcal{BLS} \text{ fair value } = e^{-rT}\mathbb{E}[\max(S_T - K, 0)]$$

where r is the interest rate and the convention is that the adjustment for interest is done with r being the continuously compounded rate. For certain technical reasons that need not concern us here, we have to set $\mu = r - q - \frac{1}{2}\sigma^2$ to get the right fair value under the model, where q is the dividend yield on the stock. (The dividend yield on the market index SPY is around a nice steady, 2.10% per year, or $q = 0.0210$.)

The expected value of the BLS computation above simply involves a lot of college-level calculus that need not concern us here. (A reference that explicitly shows all the calculus is the book Stoll and Whaley entitled "Futures and Options: Theory and Applications"). For our purposes all you need to know is that the

$$\text{fair value of call} = \mathcal{BLS}\,(S_0, K, T, r, \sigma, q)$$

where $\mathcal{BLS}$ is the Black-Scholes call pricing formula. There are many online calculators that give $\mathcal{BLS}$ once the inputs are entered. In addition, it is easy to implement the function and it is already implemented in various packages for different programming languages.

2 The Importance of Units of Measurement

We do have to be very careful about the units of measurement, or the $\mathcal{BLS}$ function will give ridiculous numbers. In financial econometrics, the unit of time is a day, and we measure rates like returns, the interest rate, etc. as a percent, for example an interest rate might be 4.25% per year, or annualized volatility might be 21.23% per year.

In options analysis and derivatives, the unit of time is one year and annualized rates are usually expressed as decimal numbers. Specifically,

$S_0 =$ current stock price in dollars, e.g., 57.42 dollars.

$K =$ strike price in dollars, e.g., 57.50 dollars.

$r =$ interest rate as decimal number, e.g., 1.75% $\Rightarrow r = 0.0175$

$T =$ expiration in years, e.g., 38 days $\Rightarrow$ T = 38/365

$\sigma =$ annualized volatility expressed as a decimal number, e.g., 16.55% per year, would be $\sigma = 0.1655$

$q =$ dividend yield as decimal number, e.g., 2.31% $\Rightarrow q = 0.0231$

Users need to be careful because some online $\mathcal{BLS}$ calculators use percent instead of decimal numbers. Read the documentation.

3 Using the BLS Formula

To implement the formula we need only have values for the five input parameters. Four of these are easy, while the volatility needs to be determined. The inputs are

$$\begin{aligned} S_0 &= \text{Known} \\ K &= \text{Known characteristic of the option} \\ r &= \text{Known, use a Federal Reserve web site, e.g., FRED} \\ T &= \text{Known characteristic of the option} \\ \sigma &= \boxed{\;??\;} \quad \text{What to use?} \\ q &= \text{Known, use one of many web sites, e.g., } \texttt{dividend.com} \end{aligned}$$

We have been measuring volatility all semester, and it is known that the high frequency data provide the best information about asset volatility. In the next project you will investigate how well using annualized realized volatility works to implement the BLS formula, in the sense that you will plug RV (in appropriate units) into the BLS calculator to see how closely the predicted model-implied option price is to the observed market price.

4 Put Options

Everything above applies to the call option, because it is a bit simpler to think about the right to buy an asset; a put option is just the opposite of a call option. A put is the right to sell the asset at strike K. At expiration T, we have

$$\text{Payoff of put option at } T\text{: } \max(K - S_T, 0)$$

The payoffs (values) of the call and put with the same strikes at expiration can be summarized as

Value of stock:	$S_T < K$	$S_T = K$	$S_T > K$
Value of call:	0	0	$S_T - K$
Value of put:	$K - S_T$	0	0

We can always use the stock itself plus one type of option to synthesize the other option. For example, consider a portfolio comprising one share of stock and one put option. The portfolio's payoff at time T is

$$S_T + \max(K - S_T, 0) = \max(K, S_T) = \max(0, S_T - K) + K.$$

The left-most side is the payoff from stock+put while the right-hand side is the payoff of the call+K. The stock and put together are equivalent to a call plus cash. In the absence of dividends the put price p and call price c are connected via the parity relationship

$$p + S_0 = c + PV(K)$$

where $PV(K) = e^{-rT}K$ is the present value of the strike. The presence of dividends makes the parity condition a little more involved:

$$p + e^{-qT}S_0 = c + e^{-rT}K$$

To compute the BLS value of a put option, one can do all the integrals or simply use the parity condition; either way has to give the same answer or there is a mistake. We write the put value as

$$\text{BLS}_{put} = \text{BLS}_{call} + e^{-rT}K - e^{-qT}S_0$$

5 The Strike-to-Underlying Ratio

For reasons that will be clearer later, we often consider the ratio of the strike price K to the stock price S, instead of the difference. At expiration, if $K/S_T < 1$, then the put finished out of the money (worthless) while the call finished in the money (has value). On the other hand, if at expiration $K/S_T > 1$, then the put finished in the money (has value) while the call finished out of the money and is worthless. If $K/S_t = 1$ then both options finished right at the money and both have value zero.

At any point in time, we compare the current stock price to the strike price to label an option as either in or out of the money. We would say

$$\text{Strike-to-Underlying: } \frac{K}{S_0} < 1 \Rightarrow \text{put out of the money, call in the money}$$
$$\text{Strike-to-Underlying: } \frac{K}{S_0} > 1 \Rightarrow \text{put in the money, call out of the money}$$

Obviously, whether the option is in or out of the money varies over time depending upon the strike-to-underlying ratio, which itself varies over time as the stock price fluctuates.