# Project 4: Bootstrap and Realized Beta

## 1 Instructions

Project 4 is due on September 27th by 10:00 pm. This is a hard deadline, so no exceptions. You must push your local repository back to GitHub before the deadline. Your repository must contain:

- The Matlab code you used to complete the project;
- A script named `main.m` file that generates all required plots;
- A `report.pdf` file with your answers to the project questions. The report must also contain an Appendix with the code used to solve the project;
- All plots in the report must be self-contained. Self-contained means that a reader who only sees your figure (image and caption, but not the surrounding text) can understand what you are plotting. This translates to all plots having axis titles, correct units on the axis, and a caption that summarizes what is plotted.

This project makes use of stock data. Refer to the Data page for instructions on how to download the data and which files to download (requires Duke login). You must complete all exercises for both of your stocks using the data at the 5-minutes sampling frequency, unless stated otherwise.

You can obtain the repository for this project by clicking **on this link**.

## Questions

The purpose of this project is to learn how to conduct valid inference using the bootstrap method. You will also implement an estimator for the integrated beta and conduct inference using bootstrap.

### Exercise 1 - Bootstrapping Confidence Intervals for the Truncated Variance

On the previous project you have computed confidence intervals for the truncated variance based on the asymptotic theory. The objective of this exercise is to compute confidence intervals using the bootstrap method.

#### A.

Separate jump returns from diffusive returns using $\alpha = 4.5$. How many jumps do you observe per year? Are there periods with a higher concentration of jumps or are jumps uniformly distributed over the years?

**B.**

Estimate the truncated variance.

**C.**

Use the bootstrap method to create 95% confidence intervals for the integrated variance. Use $M = 11$. Zoom in on a 2-weeks period and plot the truncated variance and confidence intervals. Interpret.

Hint: First, test your code using a small number of bootstrap samples, like 100. Make sure your code works correctly and then scale the number up to 2000 or 10000. You will need to be efficient when coding this problem, or your program might take too long to run.

**D.**

Do you need to use the Delta-method to create confidence intervals for the annualized integrated variance? Explain.

**E.**

Compute confidence intervals for the annualized integrated variance. Zoom in on the same 2-weeks period and plot the annualized truncated variance and confidence intervals. Interpret.

**F. (Optional, PhD required)**

Perform a Monte Carlo analysis to evaluate whether the bootstrap confidence intervals are accurate with respect to the integrated variance. To do so, simulate from the model:

$$\begin{aligned} dX_t &= \sqrt{c_t} dW_t \\ dc_t &= \rho(\mu_c - c_t)dt + \sigma_c \sqrt{c_t} dW_t^c \end{aligned}$$

Where $X$ denotes the log-price of an asset, and $W$ and $W^c$ are independent.

Assume that:

$$\begin{aligned} n &= 80 \\ T &= 1.25 \times 252 \\ n_E &= 20 \times n \\ \rho &= 0.03 \\ \mu_c &= 0.011^2 \\ \sigma_c &= 0.001 \\ c_0 &= \mu_c \\ X_0 &= \log 292.58 \end{aligned}$$

Use the following additional rule. If the simulated $c$ becomes smaller than $\mu_c/2$ change the value to $\mu_c/2$ instead.

Given your previous findings (project 3), are the bootstrap confidence intervals more or less accurate? Or are they very similar to the confidence intervals computed from the asymptotic theory?

## Exercise 2 - Integrated Beta

The objective of this exercise is to estimate and conduct inference on the integrated beta. You will use the 5 minute data for the SPY (market index) in addition to your two stocks. If one of your stocks is already the SPY, then choose a third different stock from the data folder.

### A.

We want to explore the dependency of moves in a stock to moves in the market index. Estimate the realized beta for each of your stocks with respect to the market index. Plot the realized betas for all days and interpret.

Hint: What should be $X_1$ and what should be $X_2$?

### B.

Does the realized beta vary over the years? Does it seem plausible to assume a fixed beta like in the usual CAPM?

### C.

Use the bootstrap with $M = 7$ (or $k_n = 11$) to compute confidence intervals for the realized betas. Plot the realized beta alongside the confidence intervals and comment on the accuracy of intervals.

### D.

Zoom in on the plot for a month of interest. Interpret the plot and comment on the accuracy of the confidence intervals.

### E.

A stock with $\beta = 1$ is called a unit beta stock, meaning it moves one-for-one with the overall market. Many of the large-cap stocks like those in the DOW are unit beta stocks. We can use the confidence intervals for the realized betas to test whether that is true. If the confidence interval contains 1, then we say there is evidence that the stock is unit beta, at least on a given day. For each stock, compute the number of days where:

- The confidence interval contains 1;
- The confidence interval is below 1 (we say the interval $[a, b]$ is below 1 if $a < b < 1$);
- The confidence interval is above 1 (we say the interval $[a, b]$ is above 1 if $1 < a < b$);

Report the numbers above. Would you say that the stock is generally as risky as the market? Or more so, or less so? Justify.